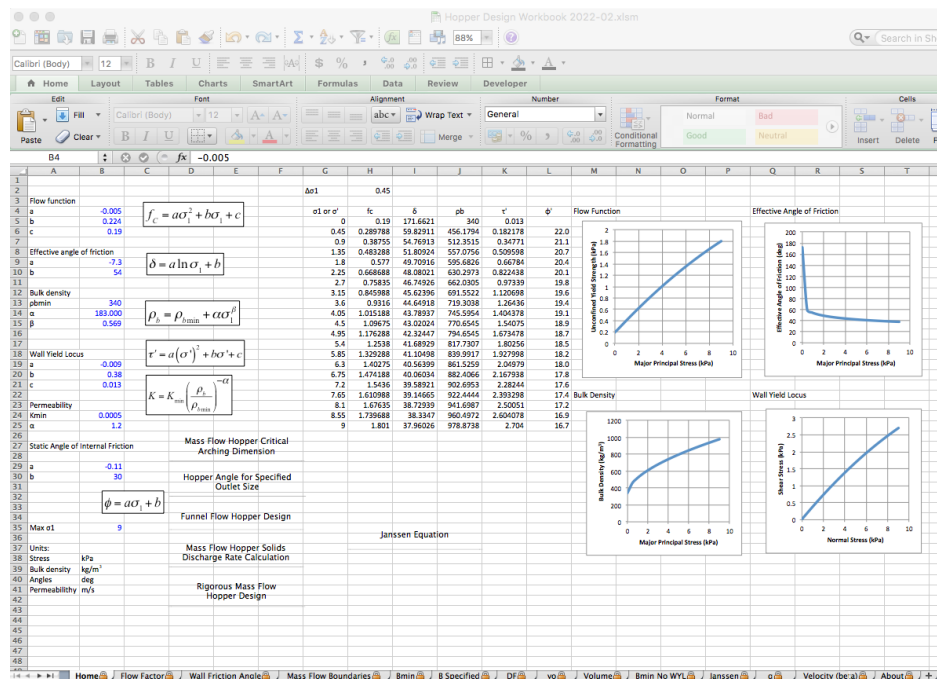


Note: My companion regression workbook is useful as it calculates the regression constants that are used in the hopper design workbook. A couple of pointers:

- Do not be fooled by R^2 values. For example, if you have three data points for the flow function to regress and you choose a quadratic model, the correlation factor will be equal to 1, but you'll likely calculate a very small arching dimension as the critical stress must be determined by extrapolation.
- Note the unique functions for the effective angle of friction and the bulk density.
- I like to use a quadratic to fit the wall friction test results.
- The value of c (the intercept) should never be negative. If you get this result, it's best to use the "Linear through Origin" or "Quadratic with Fixed Intercept" workbooks, using zero for the intercept.
- Read my textbook for advice for choosing a model for the flow function. The choice of regression model and greatly affect the calculations.

Powder flow properties input

Clicking the *Home* tab will display the following worksheet:



Powder flow property correlations are entered here. The flow function and wall yield locus are nonlinear. (Linear correlations of course can be used by setting the coefficient a equal to zero.) The effective angle of friction is linear with respect to the logarithm of the major principle stress. If regression yields a positive slope, the effective angle of friction can be assumed constant and equal to the average of the test results by setting b equal to the average value of σ_1 and a equal to zero. The bulk density is nonlinear. Note the form of its empirical relation. A constant value of the static angle of friction is used, while K uses a power-law relationship. Note that the results of the analyses can be very dependent on your choice of regression model.

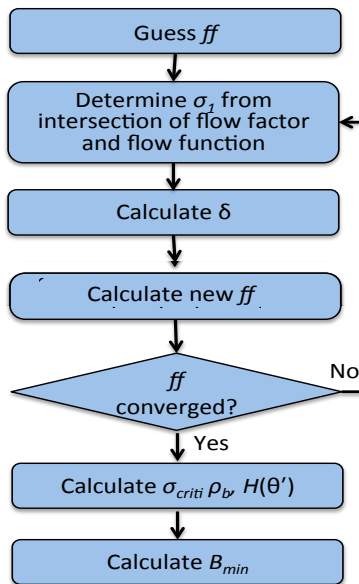
The units for stress and strength are kilopascals. Units for bulk density are kilograms per cubic meter. Angles are in degrees. Permeability is in meters per second.

The user should confirm the regression inputs and compare the plots and tabulated values of the strength, effective angle of friction, bulk density, and wall friction with those given by test results. Enter a value for the maximum major principal stress so that the plots will cover the range of stresses for which you have data.

Critical hopper outlet dimensions can be sensitive to the choice of the model used in the regression of the cohesive strength data to define the flow function. Use engineering judgment – determination of the critical stress requires extrapolation of the flow function data.

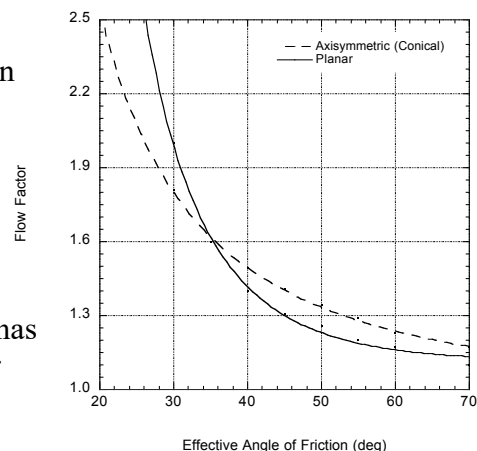
Mass flow hopper critical arching dimension

Refer to the flow chart shown below:



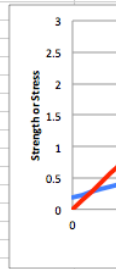
The critical arching diameter requires calculation of the critical stress, which is found by determining the intersection of the flow function (relationship between the powder's unconfined yield strength and major principal stress) and flow factor (relationship between the stress on the abutments of an arch at a hopper outlet and the major principal stress). The flow factor depends on the angle of wall friction, the geometry of the hopper, and the hopper angle, which may not be known *a priori*.

The adjacent figure plots values of the flow factor as a function of the effective angle of friction and is based on a figure provided by Jerry Johanson in Kulwiec (Materials Handling Handbook, John Wiley and Sons, Hoboken, NJ, 1985). The flow factor determined from Johanson's relation is approximately the same that would be calculated for wall friction angles greater than about 12° and hopper angles in the neighborhood of the mass flow boundary. If a powder has exceptionally low wall friction, the rigorous method for determining B_{min} should be followed.



From the *Home* tab, click the *Mass Flow Hopper Critical Arching Dimension* button. This will open the following worksheet:

	A	B	C	D	E	F	G	H
1								
2			$f_c = a\sigma_1^2 + b\sigma_1 + c$	$\rho_b = \rho_{bmin} + \alpha\sigma_1^\beta$		$\delta = a \ln \sigma + b$		
3								
4		kPa		kg/m3 vs. kPa				
5	a	-0.005		pbmin	340		Effective angle of friction	
6	b	0.224		α	183		a	
7	c	0.19		β	0.569		b	
8								
9	i	1	1 = 1 for round, 0 for slot					
10	ff	1.4						
11	θ'	19.5		Confirm $\Delta = 0$				
12				Δ	-1.0712E-05			
13	P-factor	1						
14								
15								
16				ff	1.22			
17		Conical		ff * P-factor	1.22			
18	H	2.30						
19	Bmin (m)	0.14			0.14			
20	Bmin (mm)	140.56			140.56			
21	Bmin (in.)	5.53			5.53			
22								
23								
24	Successive substitution							
25	b-1/ff	-0.49028571		σ_{1crit}	0.318076687			
26	σ_1	0.386009581		σ_{crit}	0.260743314			
27	δ	60.94881955		pbo	435.3655914			
28	ff	1.22990884		δ	62.36163239			
29	b-1/ff	-0.58906839						
30	σ_1	0.321664956						
31	δ	62.27998693						
32	ff	1.220447059						
33	b-1/ff	-0.59537188						
34	σ_1	0.31827754						
35	δ	62.35727002						
36	ff	1.219914362						
37	b-1/ff	-0.59572967						
38	σ_1	0.318087399						
39	δ	62.36163239						
40	ff	1.219884346						
41	b-1/ff	-0.59574984						
42								



An initial guess of the flow factor (default value equals 1.4) is input along with the shape factor i (enter 1 for a round outlet, 0 for a slotted outlet) and the hopper angle. (19.5° is suitable for design purposes; this value can be overridden.) A P-factor can be entered to determine the effect of vibration, impact, *etc.* on critical arching dimensions.

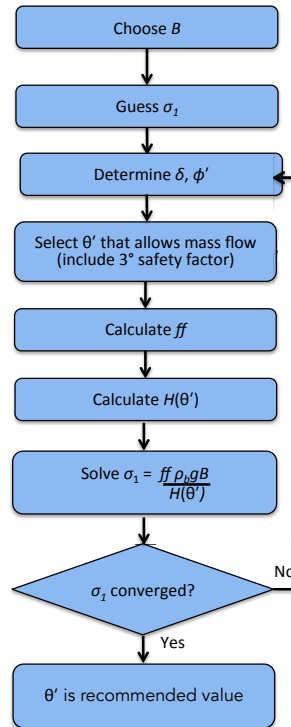
Calculation of recommended mass flow hopper angle for specified outlet size.

Refer to flow chart on the following page.

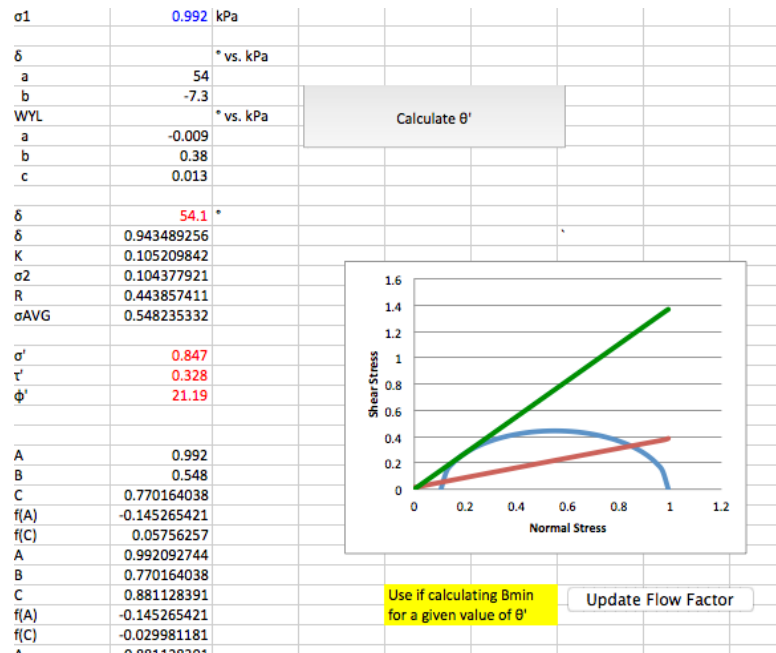
1. If on the *Home* worksheet, click the *Hopper Angle for Specified Outlet Size* button. This will bring you to the *B Specified* worksheet.

	A	B	C	D	E	F	G	H	I
	pbmin	340		$\rho_b = \rho_{bmin} + \alpha\sigma_1^\beta$					
	α	183							
	β	0.569							
	B	0.3048							
	i	1	1 = 1 for round, 0 for slot						
	σ_1	0	Confirm $\Delta = 0$	Δ	0.0024				
	ff	1.4							
	H	2.2							

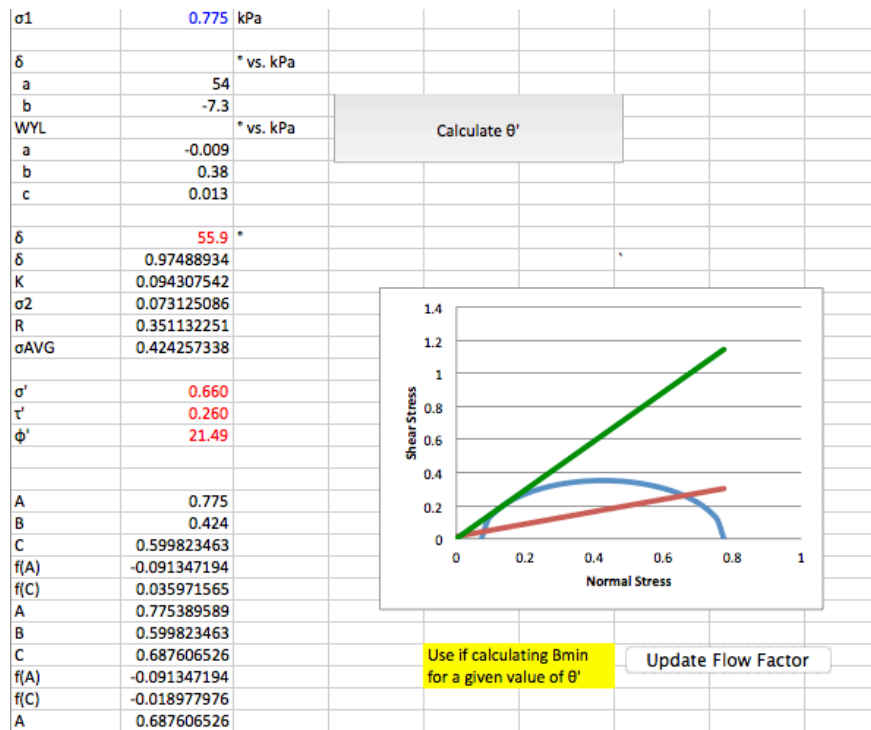
Calculate δ and ϕ'



2. Enter an initial estimate of the flow factor. (The flow factor used to calculate the minimum arching dimension can be used.) A successive substitution algorithm is used to calculate the major consolidation stress σ_1 . An initial guess is required; 0 can be used. Convergence should be confirmed by verifying that Δ is 0.
3. Click *Calculate δ and ϕ'* to open the *Wall Friction Angle* worksheet.



4. Click the *Calculate θ'* to open the *Mass Flow Angle* worksheet.

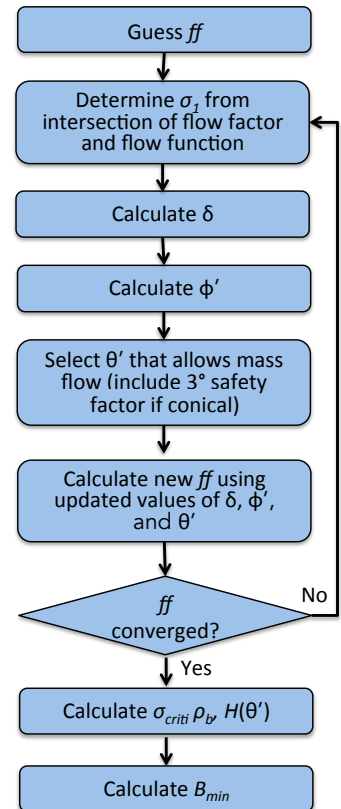


7. Note the value of recommended mass flow hopper angle. Repeat steps 3-7 until convergence has been reached.

Rigorous mass flow hopper design

Refer to flow sheet to the right:

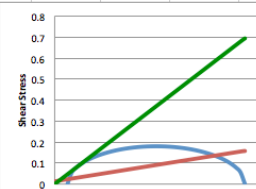
1. If on the *Home* worksheet, click the *Rigorous Mass Flow Hopper Design* button. This will bring you to the *Bmin* worksheet.
2. Any cells that are colored blue can be overridden. Enter an initial estimate for the flow function ff (1.2 – 1.4 is usually a good choice) and the geometry factor i (1 for a round outlet; 0 for a slot).
3. Optionally enter a value of the hopper angle θ .
4. Updated values of the major principle stress σ_1 at the hopper outlet and other calculation results will appear in red.
5. Click *Calculate δ and ϕ'* to open the *Wall Friction Angle* tab. The spreadsheet will calculate values of the effective angle of friction and wall friction angle.



$f_c = a\sigma_1^2 + b\sigma_1 + c$		$\rho_b = \rho_{bmin} + \alpha\sigma_1^\beta$	
	kPa		kg/m3 vs. kPa
a	-0.005	pbmin	340
b	0.224	α	183
c	0.19	β	0.569
ff	1.4		
θ'	20		
i	1	i' = 1 for round, 0 for slot	
b-1/ff	-0.49028571		
σ_{1crit}	0.386	Calculate δ and ϕ'	
σ_{crit}	0.276		

6. Click the *Calculate θ'* button. This will bring you to the *Mass Flow Boundaries* tab.

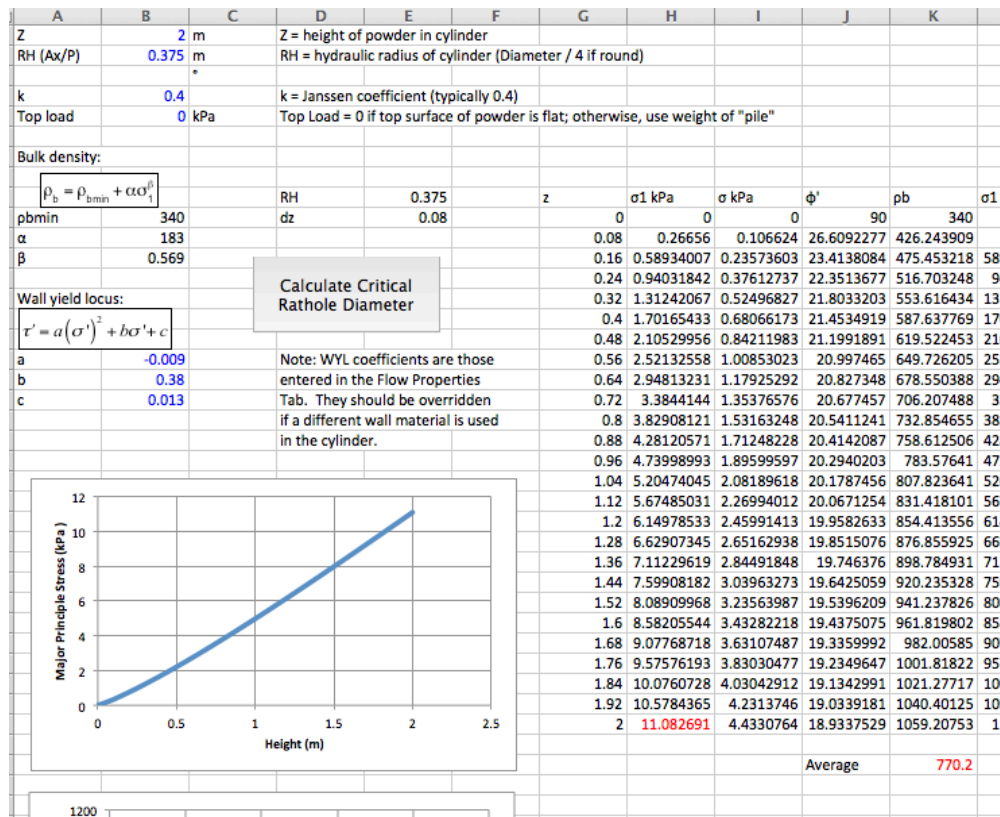
σ_1	0.386 kPa		
δ		* vs. kPa	
a	54		
b	-7.3		
WYL		* vs. kPa	
a	-0.009		
b	0.38		
c	0.013		
δ	60.9 *		
δ	1.063757576		
K	0.067129776		
σ_2	0.025912737		
R	0.180048422		
σ_{AVG}	0.205961159		
σ'	0.325		
τ'	0.135		
ϕ'	22.64		
A	0.386		
B	0.206		
C	0.29598537		



- Note the value of the recommended mass flow hopper angle and click the *Update Flow Factor* button. This will bring you to the *Flow Factor* worksheet.
- The spreadsheet will update the value of the flow factor based on new values of δ , ϕ' and θ' . Click on the *Update Flow Factor* tab. This will return you to the *Bmin* worksheet.
- Note the value of the flow factor. If it had not appreciably changed, the solution converged. Note the values of the minimum outlet diameter or width and the recommended mass flow hopper angle. If the flow factor changed appreciably, then click the *Calculate δ and ϕ'* button and repeat Steps 5 through 8 until the flow factor calculation has converged.

Janssen equation

1. Click the *Janssen* button will bring you to the *Janssen* worksheet.
2. Enter the cylinder height and hydraulic radius (Cross-sectional area divided by the perimeter).
3. The default value of the Janssen coefficient is 0.4, but it may be overridden.
4. You may replace wall yield locus regression parameters if cylinder section is different from cone.
5. You may also enter an external load, *e.g.*, the weight of the surcharge.



Calculation of critical outlet dimensions for funnel flow bins

1. On the *Home* worksheet, click the *Funnel Flow Hopper Design* button. This will bring you to the *DF* worksheet.

$f_c = a\sigma_1^2 + b\sigma_1 + c$		$\rho_b = \rho_{bmin} + \alpha\sigma_1^\beta$		$\delta = a \ln \sigma_1 + b$		G
Flow Function, kPa		pbmin	340			D
a	-0.005	α	183			
b	0.224	β	0.569			
c	0.19	δ				
ϕ_t (internal)	28.78	a	-7.3			
σ_1	11.08 kPa	b	54			
Calculate Solids Stress from Janssen Equation		$\theta_p = \left[45^\circ - 0.5 \cos^{-1} \left(\frac{1 - \sin \delta}{2 \sin \delta} \right) \right] \left[65^\circ - 0.5 \cos^{-1} \left(\frac{1 - \sin \delta}{\sin \delta} \right) \right]^{1/2}$				
$G(\phi_t)$	2.33					
	kPa					
EH	1.068 m			Flow channel angle		
				Uniaxial	10.0	
f_c	2.06			Planar	41.6	
pb	1059 kg/m ³					
DF (m)	0.463			sin δ	0.59399039	
DF (mm)	463			1-sin δ	0.40600961	
DF (in.)	18.2			δ	36.440693	

2. Click the *Calculate Solids Stress from Janssen Equation* button.
3. Enter the cylinder height, hydraulic radius, Janssen coefficient, and if necessary the additional solids loading.
4. Click the *Calculate Critical Rathole Diameter* button.
5. Read the calculated value for *DF*, the critical ratholing diameter. *DF* is the diameter of a conical hopper or the diagonal of a hopper with flat walls and a slotted outlet that will prevent the formation of a stable rathole.
6. To determine the diameter of a planar funnel flow hopper required to prevent a stable rathole from developing, open the *Bmin* worksheet and enter $ff = 1.7$, $\theta' = 30^\circ$, and $i = 0$.
7. To determine the diameter of a conical funnel flow hopper required to prevent a stable rathole from developing, enter $ff = 1.3$, $\theta' = 7^\circ$, and $i = 1$.

Note that the critical rathole diameter D_F is equal to the diameter of a slotted outlet or the diameter of the round outlet that must be exceeded to ensure that a rathole will collapse.

The material of construction in the cylinder is not necessarily the same as that of the hopper section. The appropriate wall yield locus coefficients can be entered in the *Janssen Equation* sheet.

When designing a conical funnel flow hopper, the critical arching diameter is not typically specified since the critical rathole diameter will be significantly larger than the arching diameter.

The spreadsheet also provides the flow channel angle, *i.e.*, the angle referenced from vertical that marks the boundary between flowing material and stagnant material.

Calculation of steady discharge rate from mass flow bins

Click on the v_o tab or from the *Home* sheet, click *Mass Flow Hopper Discharge Rate Calculation* to open the v_o worksheet.

A	B	C	D	E	F	G	H	I
σ_1	9	kPa	ϕ'	21				
B	0.5	m	δ	54				
			ff	1.41		Cohesive Powder		
θ'	20							
i	1	= 1 for round, 0 for slot						
					Flow function			
			Bulk density		a		-0.005	
θ'	0.34906585		α	183	b		0.224	
RH	9		β	0.569	c		0.19	
			α		K			
$\tan\theta'$	0.36397023		fc		Kmin		0.00045	
B	0.5		a		α		1.2	
pbmax	979							
pbmin	340							
Kmin	4.50E-04							
pbo	340							
A	0.29711856							
B	1450.4							
C	-1.000							
Geometry	Conical							
v_o	0.0007	m/s				v_o	0.00050	m/s
Mdot	166	kg/hr				Mdot	122	kg/hr

1. Enter the maximum solids stress in the cylinder.
2. Enter a value of B , which is the diameter of a round outlet or the width of a slotted outlet.
3. Enter the hopper angle θ' . (For funnel flow hoppers, use the flow channel angle.)
4. Enter i equal to 1 for a round outlet, 0 for a slotted outlet, and enter the appropriate outlet dimensions.

Kerry Johanson recommends using the maximum solids stress in the cylinder to calculate the bulk density at the location of the hopper where the interstitial gas pressure is a minimum.

I have found that the predicted discharge rate better matches observations by reducing the calculated value by 80 percent.

Note: the discharge rate equation for no cohesion can be found in the 9th edition of Perry's. I've modified that equation following Jerry Johanson's method to account for the cohesive strength of the powder.

To account for cohesive strength, enter the flow factor, effective angle of friction, and wall friction angle, and click *Cohesive Powder*. If necessary, change the initial estimate of the solids stress at the outlet. This generally gives a larger discharge rate, so when designing a hopper, I do not bother using this feature.

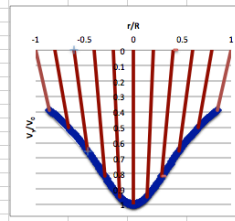
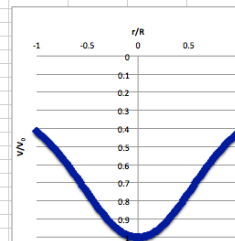
Other useful tabs

The *q* spreadsheet calculates Jenike's non-dimension surcharge factors which can be used to calculate the vertical solids stress at a hopper outlet. The wall friction angle, effective angle of friction, hopper angle, and *i* are entered.

	A	B	C	D	E	F	G	H	I	J	K	L	M
1	Jenike's non-dimensional surcharge factor												
2							$X = \frac{2' \sin \delta}{1 - \sin \delta} \left(\frac{\sin(2\beta + \theta')}{\sin \theta'} + 1 \right)$						
3													
4	<i>i</i>		1 = 1 for cone, 0 for slot										
5	ϕ'	15					$Y = \frac{[2(1 - \cos(\beta + \theta'))] \sin \theta' (\beta + \theta')^{1-i} + \sin \beta \sin^{1+i}(\beta + \theta')}{(1 - \sin \delta) \sin^{2+i}(\beta + \theta')}$						
6	δ	45											
7	θ'	20											
8													
9	ϕ'	0.26179939					$2\beta = \phi' + \sin^{-1} \left(\frac{\sin \phi'}{\sin \delta} \right)$						
10	δ	0.78539816											
11	θ'	0.34906585											
12													
13	β	0.31826691					$\frac{\sigma'}{\rho_s g B} = \frac{Y}{X-1} \left(\frac{1 + \sin \delta \cos 2\beta}{2 \sin \theta'} \right)$						
14	X	16.5967212											
15	Y	3.83977688											
16													
17	$\sigma'/(p g B)$	0.56456099					$q = \left(\frac{\pi}{3} \right)^{\frac{1}{2}} \frac{1}{4 \tan \theta'} \left[2 \left(\frac{\sigma'}{\rho_s g B} \right) (\tan \theta' + \tan \phi') - \frac{1}{1+i} \right]$						
18													
19													
20													
21	q	0.15357812		Y1	0.42904961								
22				Y2	0.34202014								
23				Y3	0.11985772								
24				Y4	0.06943146								
25				Y	3.83977688								
26													

The *Velocities* spreadsheet calculates solids velocity profiles in converging hoppers. After the wall friction angle, effective angle of friction, hopper angle, and *m* are entered, the *Calculate V/V0* button is clicked to calculate the radial and vertical solids velocity profiles.

m	1 = 1 for round, 0 for slot												
ϕ'	20												
δ	50												
θ'	20												
			β	23.2588926									
			Wall Velocity/Centerline Velocity			0.415							
ϕ'	0.349												
δ	0.873												
θ'	0.349												
β	0.406												
X	24.110				Y1	0.54347077							
Y	4.932				Y2	0.34202014							
S(0)	0.21339926				Y3	0.18545078							
					Y4	0.07529532							
Geometry	Circular				Y	4.93163063							
					$\sin \delta$	0.76604444							
$\delta \theta$	-0.0069813				$\int \tan(2\psi) d\theta$	0.30068678							
					$\psi(\theta)$	89.999835							
$\frac{d\psi}{d\theta} = -1 - \frac{m \sin \delta (1 + \sin \delta (\cot \theta \sin 2\psi + \cos 2\psi - 1) + \cos \theta - \sin \delta \cos(\theta + 2\psi) + \sin^2 \delta)}{2 \sin \delta (\cos 2\psi - \sin \delta)}$ $\frac{d\psi}{d\theta} = \frac{\sin 2\psi + \sin(\theta + 2\psi) + m \sin \delta (\cot \theta (1 + \cos 2\psi) - \sin 2\psi)}{\cos 2\psi - \sin \delta}$ $\psi(\theta) = \frac{1}{2} \left[\phi' + \sin^{-1} \left(\frac{\sin \phi'}{\sin \delta} \right) \right] + 90^\circ$ $\psi(90^\circ) = 90^\circ$ $\frac{V}{V_0} = \exp \left[-(2+m) \int_0^\theta \tan(2\psi) d\theta \right]$													



The *Volumes* spreadsheet calculates volumes, hopper angles. You can determine both the actual volume and working volume (if you have a tall enough cylinder).

Note: use the Regression Workbook to regress the flow property test data. If a negative intercept is found, fit the equation through the origin. For the flow function, consider using linear regression of the low-stress data points to determine the intercept, and then fit the data to a quadratic fixing the value of the intercept equal to that obtained from the linear regression.

Disclaimer: I make no warranty and the user should always check the outputs of the workbook with hand calculations based on Jenike Bulletin 123 and other references.